# A Variable Density Stand Level Growth and Yield Model for Even-Aged Natural Longleaf Pine

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# A VARIABLE DENSITY STAND LEVEL GROWTH AND YIELD MODEL FOR EVEN-AGED NATURAL LONGLEAF PINE

Dwight K. Lauer and John S. Kush

## INTRODUCTION

Longleaf pine (*Pinus palustris* Mill.) stands cover some 3 million widely distributed and fragmented acres in the South of which 2.7 million acres (91 percent) support natural stands and contain 94 percent of the species' growing stock volume (Outcalt and Sheffield 1996). These natural stands are a very important source of high-value wood products, provide unique multiple-use benefits, maintain biological diversity, and supply necessary habitat for several rare and endangered species. Management of longleaf pine to improve production of high value wood products and to improve a range of ecosystem services, including management of understory species composition and advancing stands towards stand structures for species such as the red-cockaded woodpecker (*Picoides borealis*), requires management of stand densities within reasonable ranges (Shaw and Long 2007). For many private landowners, longleaf will be favored only if periodic income (from hunting, pine straw production, carbon credits, habitat exchanges, and wood yields) can be demonstrated over long rotations.

The model system detailed in this publication was developed to provide growth and yield information necessary to enhance management plans that fulfill these widely diverse management objectives. Of particular importance is the use of variable density equations developed from long-term data that included multiple thinning in a structured design. This effort also updates components of previous modeling efforts (Farrar 1985, Farrar and Matney 1994) utilizing 40 years of measurement data from the Regional Longleaf Pine Growth Study (RLGS).

## GROWTH STUDY METHODS

Data used for this analysis were collected on permanent measurement plots initiated in 1964 by the U.S. Forest Service on both public and private land. The study was established to investigate production of thinned, even-aged, naturally regenerated stands in the East Gulf region of the southern United States. Plots were selected to fill an array of cells with five 20-year age classes, five 10-foot site index classes, and five 30-$ft^2$ basal area classes (Farrar 1979, 1993). Permanent circular plots were mostly 0.20 acre (with some 0.10 acre) surrounded by similarly treated 33 foot wide buffer zones. Plots were measured at five-year intervals except for a subset of plots included in a special series that were measured more frequently. Diameter at breast height (dbh) was measured on all trees and a subsample of trees was measured for height. At time of plot installation, every fifth tree in each 1 inch dbh class was selected for height measurement and cored to determine age if the tree was dominant or co-dominant. When possible, there were at least two height sample trees per dbh class and a minimum of 10 sample trees per plot, unless there were less than 10 trees in which case all trees were measured. Site index was computed with the site index equation by Lauer and Kush (2010) using the measurement closest to the base age for plots that had not yet reached base age, closest to base age for plots older than base age when established, or averaging the two site index estimates closest to base age for plots that had measurements both before and after the base age. Plot volumes were computed using volume equations by Farrar (1981) using average height for a tree's dbh class.

A total of 2009 observations on 307 plots were included in this analysis. This provided a total of 1700 non-overlapping measurement intervals or growth periods. Data from an additional 32 plots located in a contiguous area were excluded (141 intervals) because damage from frequent burning increased mortality and retarded height growth for prolonged periods resulting in unreliable estimates of site index. Many plots were subjected to hurricane damage over the history of this study but a plot growth period was excluded only if damage was severe with respect to stand conditions. Excessive hurricane or tornado damage necessitated the exclusion of 33 individual plot growth periods. Plots with less than 30 trees per acre were excluded if basal area dropped below 25 $ft^2$ $ac^{-1}$ or if the plot endured losses of more than 30 percent basal area or 30 percent of trees. Plots with 30 to 500 trees per acre were excluded if mortality reduced either stand basal area or number of trees by more than 45 percent. Plots with more than 500 trees per acre were excluded if mortality reduced both stand basal area and number of trees by more than 45 percent. Plots with severe damage were retained for future measurements if salvage

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could restore the stand to a reasonable condition in terms of tree spacing and residual basal area.

The range of data in terms of trees per acre, site index, and initial stand basal area was extensive (Figure 1). Thinning was an integral part of this study with plots receiving a target basal area of 30, 60, 90, 120, or 150 ft$^2$ ac$^{-1}$ as part of the study design. Plots that exceeded their target basal area at a given measurement by more than 7.5 ft$^2$ ac$^{-1}$ were thinned from below back to the target level. In addition, some plots were designated to receive no thinning. Initially, some plots were allowed to grow into a target basal area before thinning and others were thinned down to a target. Initial plot installation occurred across a range of ages and additional plots were added through time as the study progressed. A total of 81 percent of measured growth periods occurred after at least one thinning. Distribution of intervals by thinning target was 17 percent, 19 percent, 19 percent, 13 percent, and 13 percent for target basal areas of 30, 60, 90, 120, and 150 ft$^2$ ac$^{-1}$, respectively. Distribution of growth periods by target basal area class and number of times thinned is listed in Table 1.

## GROWTH AND YIELD MODEL SYSTEM

A modification of the Clutter and Jones (1980) model was used to project longleaf pine basal area growth. The yield model from which this path invariant projection equation was derived is:

$$B = \theta^{(1/(A+\gamma_3))^{\gamma_0+\gamma_1 ln(N)}} exp(\gamma_2). \qquad (1)$$

The significant modification is that of the exponent associated with the age (A) term. Modifications of this type were done by Pienaar et al. (1985) with a thinning index function and by Rayamajhi (1996) with a function of climate variables to account for changes in growth rate. In this case, a function of the natural log of number of trees (N) was used to construct a variable density model to describe the effects of thinning regimes.

Figure 1. Range of study data for trees per acre (top), site index (middle), and stand basal area (bottom) versus breast height age (years) of dominant and co-dominant trees. Site index is base age 50 (43 breast height age + 7). Patterns with basal area are due to repeated thinning to target basal areas.

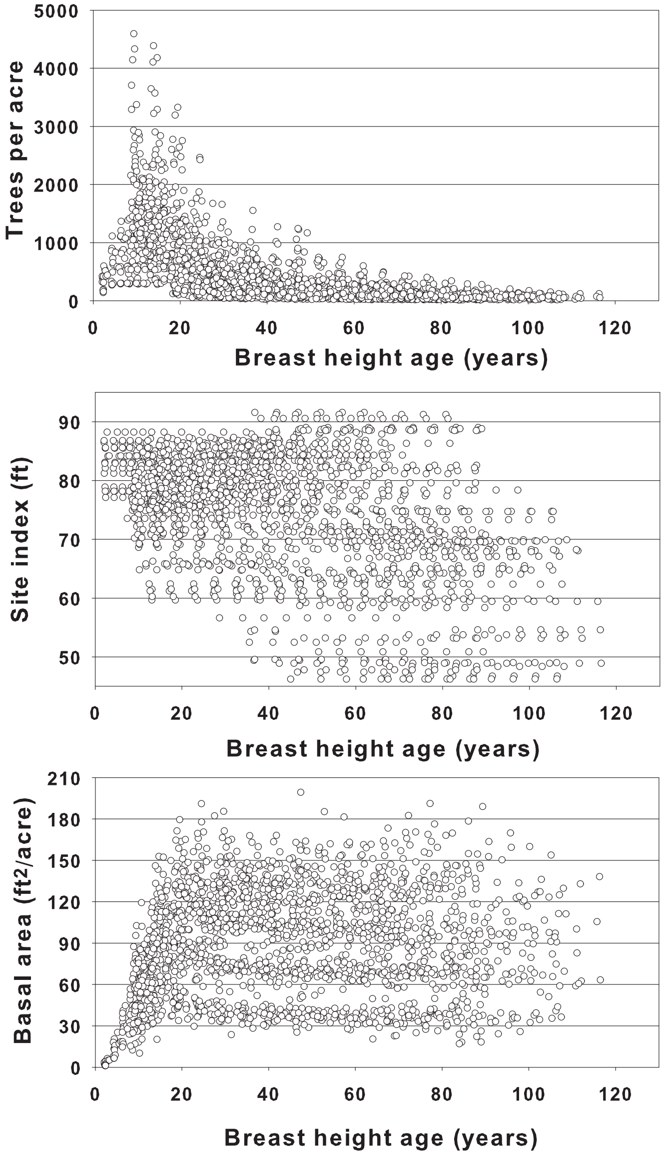


**Table 1. Distribution of Measurement Plot Growth Periods by Basal Area Thinning Class and Number of Previous Thins**

| Thinning class (ft$^2$ ac$^{-1}$) | Number of thins | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
| 30 | 27 | 57 | 84 | 65 | 52 | 17 | 10 | 2 | | 314 |
| 60 | 43 | 75 | 90 | 71 | 40 | 30 | 13 | 5 | 3 | 370 |
| 90 | 42 | 85 | 88 | 69 | 36 | 23 | 13 | 4 | 1 | 361 |
| 120 | 80 | 85 | 66 | 39 | 17 | 13 | 4 | 1 | | 305 |
| 150 | 130 | 100 | 90 | 18 | 10 | 2 | | | | 350 |
| Total | 322 | 402 | 418 | 262 | 155 | 85 | 40 | 12 | 12 | 1700 |

The basal area projection model is:

$$B_2 = B_1^{\left[\frac{(A_1+\gamma_3)^{R_1}}{(A_2+\gamma_3)^{R_2}}\right]} exp\left[\gamma_2\left(1-\frac{(A_1+\gamma_3)^{R_1}}{(A_2+\gamma_3)^{R_2}}\right)\right], \qquad (2)$$

with $R_1 = \gamma_0 + \gamma_1 ln(N_1)$ and $R_2 = \gamma_0 + \gamma_1 ln(N_2)$.

$B_1$ is stand basal area ($ft^2$ $ac^{-1}$) at the beginning of the projection period, after thinning if thinning occurs. $B_2$ is stand basal area at the end of the projection period before thinning if thinning occurs. $A_1$ and $A_2$ are breast height age (years) of the dominant and co-dominant trees at the beginning and end of the projection period, respectively. $N_1$ and $N_2$ are number of trees per acre in the regeneration cohort (including trees below 4.5 feet in height) at the beginning and end of the projection period, respectively. Parameters $\gamma_0$, $\gamma_1$, $\gamma_2$, and $\gamma_3$ are to be determined, *exp* is the natural exponential function, and *ln* denotes the natural log function.The parameter $\gamma_3$ was added because breast height age is used instead of stand age.

Mortality has been observed to increase with site index and decrease in thinned stands. A stand level survival equation based on Clutter and Jones (1980) was modified to account for site index with a modification similar to that by Pienaar et al. (1990), except that the site index modifier (S in feet) is in the exponential term:

$$N_2 = \left[N_1^{\left(\lambda_1+\frac{\lambda_2}{S}\right)} + \lambda_3\left(\left(\frac{A_2}{200}\right)^{\lambda_4} - \left(\frac{A_1}{200}\right)^{\lambda_4}\right)\right]^{1/\left(\lambda_1+\frac{\lambda_2}{S}\right)} \qquad (3)$$

The following system of equations was developed to use with equations (2) and (3) when fit as a system of equations as described by Borders (1989) and using SAS Proc Model (SAS Institute 2004):

$$VOB_1 = exp\left[\alpha_0 + \frac{\alpha_1}{A_1} + \alpha_2 \ln(H_1) + {}^{\alpha_3}/_{S} + \alpha_4 ln(N_1) + \alpha_5 \frac{ln(N_1)}{A_1} + \alpha_6 ln(B_1)\right]$$

$$VOB_2 = exp\left[\ln(\mathrm{VOB}_1) + \alpha_1\left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \alpha_2(\ln(\mathrm{H}_2) - \ln(H_1)) + \alpha_4\big(ln(N_2) - ln(N_1)\big) + \alpha_5\left(\frac{ln(N_2)}{A_2} - \frac{ln(N_1)}{A_1}\right) + \alpha_6\big(\ln(B_2) - ln(B_1)\big)\right]$$

$$(4)$$

$$VIB_1 = exp\left[\beta_0 + \frac{\alpha_1}{A_1} + \beta_2 \ln(H_1) + {}^{\beta_3}/_{S} + \beta_4 ln(N_1) + \beta_5 \frac{ln(N_1)}{A_1} + \beta_6 ln(B_1)\right]$$

$$VIB_2 = exp\left[ln(VIB_1) + \beta_1\left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \beta_2(\ln(\mathrm{H}_2) - \ln(H_1)) + \beta_4\big(ln(N_2) - ln(N_1)\big) + \beta_5\left(\frac{ln(N_2)}{A_2} - \frac{ln(N_1)}{A_1}\right) + \beta_6\big(\ln(B_2) - ln(B_1)\big)\right]$$

$VOB_1$ and $VOB_2$ are volume ($ft^3$ $ac^{-1}$) outside bark at the beginning and end of the projection period, respectively. Similarly, $VIB_1$ and $VIB_2$ are volume ($ft^3$ $ac^{-1}$) inside bark at the beginning and end of the projection period. $H_1$ and $H_2$ are the average height of dominant and co-dominant trees (ft) at the beginning and end of the projection period. S is site index (ft) using a base age of 50 (43 + 7) years assuming seven years to reach dbh which is equivalent to the average height of dominant and co-dominant trees when they have a breast height age of 43 years. This system (4) was fit with the inclusion of the basal area projection equation (2) with known $N_2$ for the entire dataset including plots considered viable after hurricane damage. This system will be useful when $N_2$ is known from inventory or predicted by an alternative survival prediction method not yet developed. Fit statistics for these equations are listed in Table 2 with parameter estimates listed in Table 3. A second system that includes equation (2) as well as the survival prediction equation (3) to predict $N_2$ was fit with hurricane damage projection intervals excluded. Hurricane related mortality was excluded because it is a discrete event that is not accurately predicted by survival equation (3). Fit statistics for these equations are listed in Table 4 with parameter estimates listed in Table 5.

**Table 2. Fit Statistics for Equations (2) and (4) Using 1,700 Observations, $N_2$ Known**

| Equation | Root MSE | R-square |
|---|---|---|
| $B_2$ | 4.2 | 0.9879 |
| $VOB_1$ | 59.2 | 0.9987 |
| $VOB_2$ | 151.8 | 0.9911 |
| $VIB_1$ | 45.2 | 0.9989 |
| $VIB_2$ | 121.5 | 0.9914 |

**Table 3. Parameter Estimates for Equations (2) and (4) Using 1,700 Observations, $N_2$ Known**

| Equation type | Parameter | Estimate | Approx. std. error | Approx. prob. >\|t\| |
|---|---|---|---|---|
| Basal area | $\gamma_0$ | 0.0193 | 0.0119 | 0.3604 |
| | $\gamma_1$ | 0.1765 | 0.0039 | 0.0001 |
| | $\gamma_2$ | 5.7390 | 0.0224 | 0.0001 |
| | $\gamma_3$ | 4.7981 | 0.2155 | 0.0001 |
| Volume o.b. | $\alpha_0$ | 0.2731 | 0.0533 | 0.0001 |
| | $\alpha_1$ | 2.8443 | 0.2705 | 0.0001 |
| | $\alpha_2$ | 0.7627 | 0.0103 | 0.0001 |
| | $\alpha_3$ | -4.5960 | 0.5241 | 0.0001 |
| | $\alpha_4$ | -0.0306 | 0.0017 | 0.0001 |
| | $\alpha_5$ | -0.4428 | 0.0304 | 0.0001 |
| | $\alpha_6$ | 1.0584 | 0.0030 | 0.0001 |
| Volume i.b. | $\beta_0$ | -0.2461 | 0.0553 | 0.0001 |
| | $\beta_1$ | 3.8226 | 0.2800 | 0.0001 |
| | $\beta_2$ | 0.8277 | 0.0107 | 0.0001 |
| | $\beta_3$ | -4.0799 | 0.5398 | 0.0001 |
| | $\beta_4$ | -0.0620 | 0.0017 | 0.0001 |
| | $\beta_5$ | -0.6482 | 0.0318 | 0.0001 |
| | $\beta_6$ | 1.0939 | 0.0030 | 0.0001 |

**Table 4. Fit Statistics for Equations (2), (3), and (4) Using 1,439 Observations, $N_2$ Predicted**

| Equation | Root MSE | R-square |
|---|---|---|
| $B_2$ | 5.6 | 0.9788 |
| $VOB_1$ | 55.5 | 0.9988 |
| $VOB_2$ | 217.7 | 0.9823 |
| $VIB_1$ | 43.0 | 0.9989 |
| $VIB_2$ | 172.6 | 0.9831 |

**Table 5. Parameter Estimates for Equations (2), (3), and (4) Using 1,439 Observations, $N_2$ Predicted***

| Equation type | Parameter | Estimate | Approx. std. error | Approx. prob. >\|t\| |
|---|---|---|---|---|
| Basal area | $\gamma_0$ | 0.2826 | 0.0310 | 0.0001 |
| | $\gamma_1$ | 0.1318 | 0.0078 | 0.0001 |
| | $\gamma_2$ | 5.5090 | 0.0209 | 0.0001 |
| | $\gamma_3$ | 2.4314 | 0.2687 | 0.0001 |
| Survival | $\lambda_1$ | -2.9703 | 0.1262 | 0.0001 |
| | $\lambda_2$ | 82.7590 | 5.0080 | 0.0001 |
| | $\lambda_3$ | 0.00191 | 0.00054 | 0.0005 |
| | $\lambda_4$ | 3.9632 | 0.1320 | 0.0001 |
| Volume o.b. | $\alpha_0$ | 0.3432 | 0.0569 | 0.0001 |
| | $\alpha_1$ | 3.1264 | 0.2845 | 0.0001 |
| | $\alpha_2$ | 0.7408 | 0.0111 | 0.0001 |
| | $\alpha_3$ | -5.7865 | 0.5537 | 0.0001 |
| | $\alpha_4$ | -0.0352 | 0.0018 | 0.0001 |
| | $\alpha_5$ | -0.4912 | 0.0324 | 0.0001 |
| | $\alpha_6$ | 1.0723 | 0.0032 | 0.0001 |
| Volume i.b. | $\beta_0$ | -0.1941 | 0.0593 | 0.0011 |
| | $\beta_1$ | 4.2479 | 0.2966 | 0.0001 |
| | $\beta_2$ | 0.8095 | 0.0116 | 0.0001 |
| | $\beta_3$ | -5.0444 | 0.5737 | 0.0001 |
| | $\beta_4$ | -0.0661 | 0.0018 | 0.0001 |
| | $\beta_5$ | -0.7125 | 0.0341 | 0.0001 |
| | $\beta_6$ | 1.1067 | 0.0032 | 0.0001 |

*Hurricane mortality not included

The intention of the model system is to use inventory data at period 1 to project growth, but when inventory data are not available, a stand level basal area model is employed to provide initial basal area. The model is:

$$B = exp\left[\delta_0 + \delta_1 \ln(H) + \frac{\delta_2}{A} + \frac{\delta_3}{S} + \delta_4 \ln(N)\right]. \quad (5)$$

Equation (5) parameters are listed in Table 6.

The use of equation system (4) requires a height projection equation. A projection equation can be derived from a site index equation developed by Lauer and Kush (2010). This equation was fit to this same dataset, accounted for serial correlation, and was constructed to use breast height age. The projection equation is:

$$H_{t+q} = (H_t - 4.5)\left[\frac{X_0 + b_2(A_t - g)^{-b_3}}{X_0 + b_2\left(A_{t+q} - g\right)^{-b_3}}\right] + 4.5, \quad (6)$$

$$X_0 = 0.5\left(H_t - 4.5 - b_1 + \sqrt{(H_t - 4.5 - b_1)^2 + 4b_2(H_t - 4.5)(A_t - g)^{-b_3}}\right).$$

The parameters are $b_1$=77.080, $b_2$=1723.39, and $b_3$=1.235. For this equation, dominant height at time t, $H_t$, is projected q years. Variables account for the use of breast height age where $H_t$ is the total dominant height at the beginning of the projection period, $A_t$ is stand age, and ($A_t$-g) is breast height age in years with g years from stand establishment to attainment of breast height. Note that stand age is not required to use initial height, the length of the projection period, and breast height age with this equation.

**Table 6. Parameter Estimates for Basal Areas Prediction Equation (4) Using 1,700 Observations***

| Parameter | Estimate | Approx. std. error | Approx. prob. >\|t\| |
|---|---|---|---|
| $\delta_0$ | -8.6810 | 0.2731 | 0.0001 |
| $\delta_1$ | 2.1890 | 0.0494 | 0.0001 |
| $\delta_2$ | 5.4638 | 0.5778 | 0.0001 |
| $\delta_3$ | 77.6810 | 2.7961 | 0.0001 |
| $\delta_4$ | 0.4947 | 0.0047 | 0.0001 |

*Root mean square error = 2.74; R-square = 0.904

Thinning in this study was performed as selection from below but was performed for a wide range of age, basal area, and site index conditions. The thinning intensity in terms of percent of trees removed could be high, especially for younger ages (Table 7). The use of the survival and basal area models to account for thinning requires an estimate of the basal area remaining after removal of a given number of trees or an estimate of number of trees remaining after the removal of a given amount of basal area. If both number of trees and basal area removed are known (as from a pre-thin sample to construct a stand table for example), the basal area projection equation (2) can be used directly with this information. A variation of a model used by Pienaar and Rheney (1993) that relates the ratio of basal area and number of trees removed in thinning was used here. The exponential term was expanded as a function to account for the range of ages, densities, sites, and prior thinning conditions represented in this study. If the level of thinning is determined by the removal of a given basal area, then the number of trees removed is estimated as follows:

$$\frac{N_r}{N_b} = \left(\frac{B_r}{B_b}\right)^{\theta_1\left(\frac{H}{QD_b}\right)+\frac{\theta_2}{QD_b}+\theta_3 QD_b+\theta_4 I_1 QD_b} . \qquad (7)$$

If the level of thinning is determined by the removal of a given number of trees, then the basal area removed is estimated as follows:

$$\frac{B_r}{B_b} = \left(\frac{N_r}{N_b}\right)^{\varphi_0+\frac{\varphi_1}{QD_b}+\frac{\varphi_2}{A}+\frac{\varphi_3}{H}+\varphi_4\left(\frac{N_a}{N_b}\right)+\varphi_5 I_1\left(\frac{N_a}{N_b}\right)} . \qquad (8)$$

In equations (7) and (8), $N_b$, $N_a$, and $N_r$ are the number of trees per acre before thinning, after thinning, and removed by thinning, respectively; $B_b$ and $B_r$ are basal area ($ft^2$ $ac^{-1}$) before thinning and removed by thinning, respectively; $I_1$=1 if this is the first time the stand has been thinned, 0 otherwise, $QD_b$ is quadratic mean dbh before thinning, and H is dominant height at time of thinning. A total of 816 thinning observations were used to obtain the parameters for equations (7) and (8) listed in Table 8.

## DISCUSSION

A preliminary attempt was made to develop a basal area projection equation with thinning indices using a Clutter and Jones type model (Bailey and Ware 1983, Pienaar et al. 1985) or a Schumacher type variable density model (Pienaar and Shiver 1986) with and without thinning indices. These equations did not perform well across all basal area thinning regimes included in this study series, particularly for the lower basal area targets of 30 and 60 $ft^2$ $ac^{-1}$. Another approach by Farrar and Matney (1994) used a ratio of quadratic mean dbh in their survival equation and separate models of thinned and unthinned stands to project arithmetic and quadratic mean dbh but noted that mortality after thinning was not highly correlated with stand parameters. The construction of separate models for thinned and unthinned stands, or a thinning index based on number of trees per unit area or ratio of quadratic mean dbh for repeatedly thinned stands was also problematic since only the last thinning event is considered or some complex function of all past thinning must be constructed.

Model (2) is a variable density basal area model that was developed to overcome the need for a thinning index to describe previous thinning history but also indicates that future basal area growth is the same for a given initial basal area, trees per acre, and age regardless of previous thinning history. This was supported in the preliminary analysis when indices to describe intensity of thinning were not successful. Methods to differentiate growth of plots that grew into a target basal area from growth of those that were thinned back to a target basal area were also unsuccessful. Farrar (1968) noted in the Loxley thinning study that dbh increment increased with decreasing residual basal area and that volume production increased with increasing residual basal area, but that there was no discernable difference in longleaf volume production related to thinning intensity. Quicke et al. (1994) developed an individual tree basal area growth model using data from this same RLGS study that described growth based on age, competitive position

**Table 7. Distribution of 816 Thinning Events by Age Class and Percent of Trees Removed by Thinning**

| Breast ht. | –Percent of trees removed by thinning– | | | | | |
|---|---|---|---|---|---|---|
| age class (years) | 20 or less | 21-40 | 41-60 | 61-80 | more than 80 | Total |
| 20 or less | 48 | 24 | 48 | 30 | 8 | 158 |
| 21-40 | 58 | 162 | 59 | 15 | 8 | 302 |
| 41-60 | 84 | 70 | 18 | 9 | 9 | 190 |
| 61-80 | 75 | 33 | 8 | 8 | 1 | 125 |
| 81 or more | 32 | 9 | 0 | 0 | 0 | 41 |
| Total | 297 | 298 | 133 | 62 | 26 | 816 |

**Table 8. Parameter Estimates for Equations (7) and (8) Using 816 Thinning Events***

| Equation | Parameter | Estimate | Approx. std. error | Approx. prob. >\|t\| |
|---|---|---|---|---|
| Eq. 7 | $\theta_1$ | -0.0289 | 0.0031 | 0.0001 |
| | $\theta_2$ | 2.4320 | 0.1375 | 0.0001 |
| | $\theta_3$ | 0.0734 | 0.0014 | 0.0001 |
| | $\theta_4$ | -0.0200 | 0.0015 | 0.0001 |
| Eq. 8 | $\varphi_0$ | 2.3345 | 0.0662 | 0.0001 |
| | $\varphi_1$ | 10.0369 | 0.3630 | 0.0001 |
| | $\varphi_2$ | -16.1889 | 1.0183 | 0.0001 |
| | $\varphi_3$ | -51.4529 | 4.1841 | 0.0001 |
| | $\varphi_4$ | -1.2559 | 0.0642 | 0.0001 |
| | $\varphi_5$ | 0.3029 | 0.0553 | 0.0001 |

*Equation (7) predicts the ratio of trees removed to number of trees before thinning (root mean square error = 0.073 and R-square=0.876). Equation (8) predicts the ratio of basal area removed to the basal area before thinning (root mean square error=0.046 and R-square=0.903).

in the stand, and tree dbh without requiring any explicit term for thinning history and found no consistent pattern in residuals related to either percent or actual basal area removed.

Model fit statistics provide some guidance in model development, but an important criterion for fitting models with this dataset was that averaged residuals be close to zero across both age and thinning classes. The top graph in Figure 2 shows a reasonable pattern of residuals for model (2) and the cluster pattern indicative of this study due to study basal area thinning targets. Box plots in Figure 2 indicate that residuals were reasonable centered about zero across both thinning target basal area classes and age classes. Stands less than 20 years breast height age are in a period during which basal area growth changes rapidly and, as expected, had higher variation. There were a number of outliers but the majority of these had a projection period with mortality (mostly hurricane related mortality) with outlier observations balanced between over- and under-prediction. Figure 3 compares basal area projection residuals in terms of percent prediction error by age class and basal area target classes when the survival model was included in the projection system ($N_2$ predicted). Basal area prediction with the survival model included performed reasonably well across age and basal area thinning classes in terms of averages being close to zero for each class. The true evaluation of this model can only be performed using an independent dataset that does not exist. The alternative would be to fit the models to a subset of data and perform tests using the excluded data. Although there are many observations, the survival model that includes site index as well as density requires a large number of observations. There is also such a wide range of density, site index, and age that modeling subsets of data will degrade the model in unknown ways. The dilemma is that such a test would provide statistics on a model that deviates from the final model in unknown ways.

As noted in previous works with this dataset (Farrar 1985, Rayamajhi 1996, Quicke et al. 1994), site index is not significant in basal area projection equations. Site index is important for growth such as in equation (5), but cancels as a constant in the projection equation because it is already accounted for by stand age, basal area, and number of trees. Equation (5) should be used with some caution because it did not perform as well for thinning classes of 30 and 150 $ft^2$ $ac^{-1}$ as it did for no-thin and other thinning classes. This model was a compromise to provide initial basal area when not available because 1) the fact that basal area is being predicted means that target basal area of thinning is not known, and 2) this model formulation violates the knowledge gained from the development of model (2) that Schumacher-type equations did not describe some of the thinning regimes as well as desired. A model using target basal area as an indicator function was not adopted because target basal area alone could be a reasonable estimate of basal area for many plots.

Figure 2. Examination of residuals (1700 observations) for predicted basal area ($B_2$) from basal area projection model (2) when $N_2$ is known. Residuals plotted against initial basal area (top) show clusters of points caused by the planned thinning regime. Box plots of residuals were used to examine the ability of the model to explain basal area growth across basal area thinning targets (middle) and age classes (bottom). Middle line is the median, dotted line is the mean, box edges are the 25th and 75th percentiles, whiskers are the 10th and 90th percentiles, and dots are the 5th and 95th percentiles.

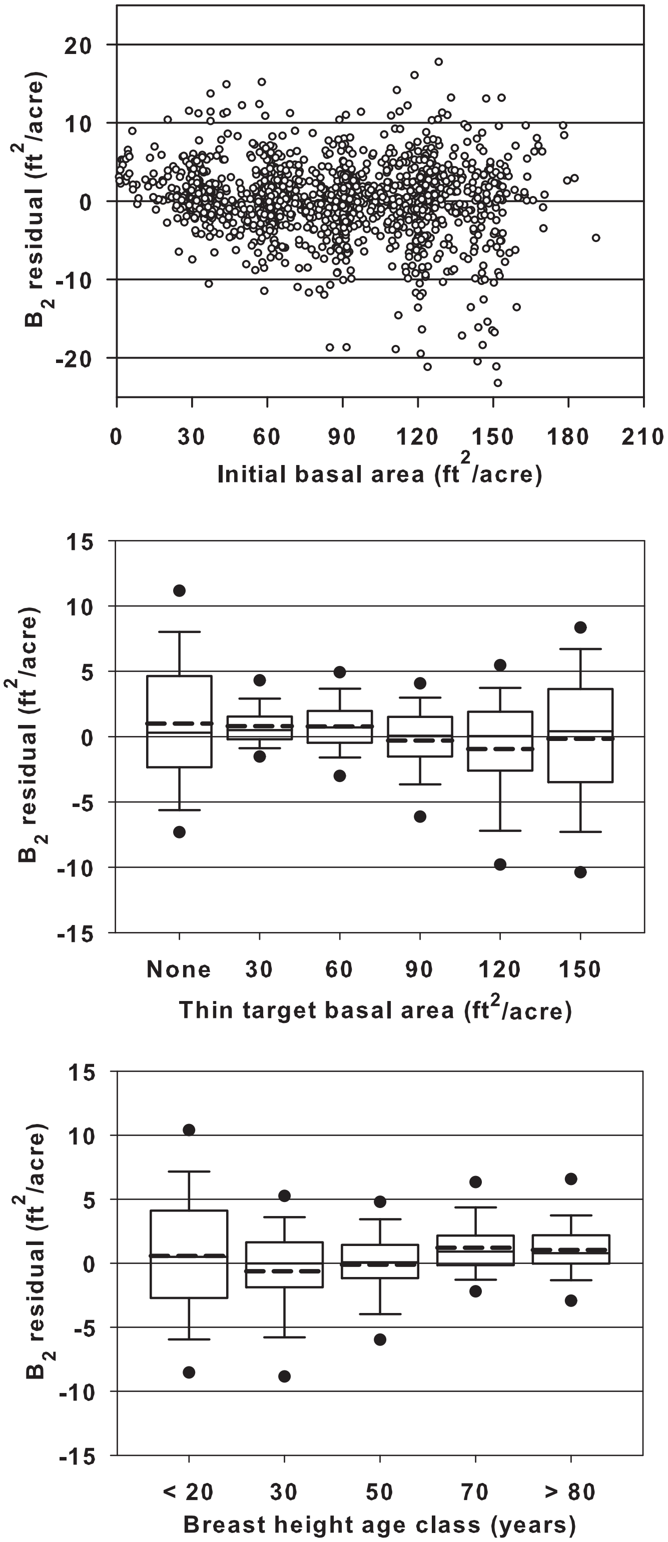
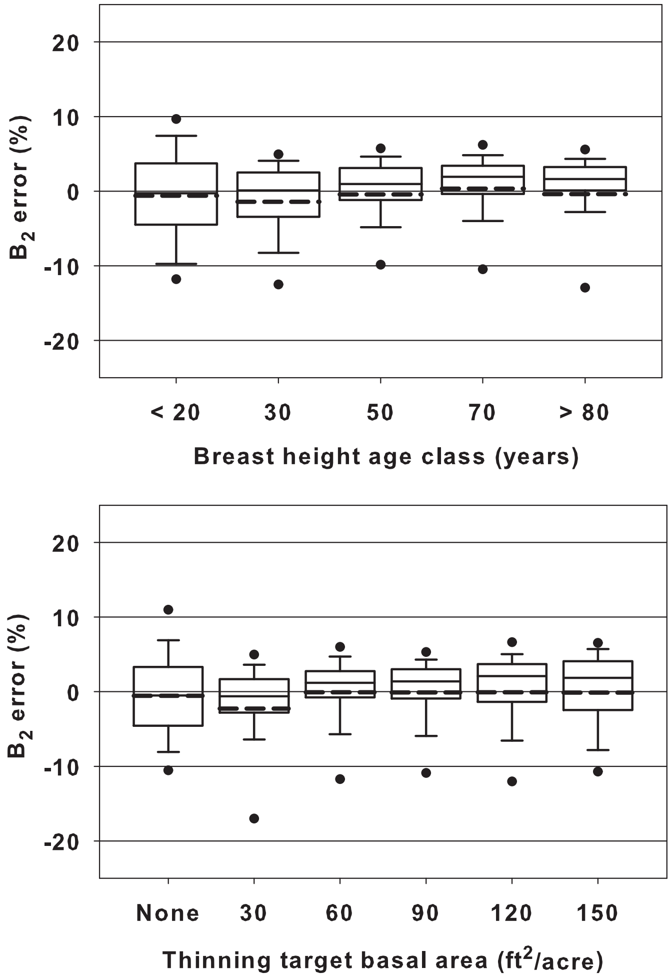


Figure 3. Percent prediction error for residuals of predicted basal area ($B_2$) when $N_2$ is predicted with a survival model using 1439 observations. Box plots of residuals were used to examine the ability of the model to explain basal area growth across basal area age classes (top) and target thinning classes (bottom). Middle line is the median, dotted line is the mean, box edges are the 25th and 75th percentiles, whiskers are the 10th and 90th percentiles, and dots are the 5th and 95th percentiles.



Equation system (4) provides inside and outside bark initial volume prediction and volume projection equations. These are variable density equations with reasonable patterns of residuals by thinning class and age class as demonstrated by the outside bark equation residuals in Figure 4. As expected, variation of residuals was higher for the projection equations, with the highest variation for plots in the 150 ft$^2$ ac$^{-1}$ thinning class. Plots in this highest basal area class are not often thinned and sometimes have difficulty maintaining this density of basal area due to mortality. Residuals from volume predictions when survival prediction is included in the model system are reasonably centered near zero. The boxplots of residuals for percent prediction error for outside bark volume in Figure 5 compare residuals by age class and thinning basal area class.

Survival is difficult to model because mortality is a relatively rare event. Mortality is due to many causes in longleaf pine stands including competition, prescribed fire, lightning strikes, tornadoes, and hurricanes. Other researchers have developed survival models that are consistent with the perception that survival rate decreases with increasing site index and is higher after thinning. Farrar and Matney (1994) developed models for thinned and unthinned stands and included a height divided by age term to account for site. Quicke et al. (1997) developed an individual tree survival model in which there was a constant survival rate (or constant mortality rate) term regardless of tree size and then a multiplier term that decreased survival with increasing competition (basal area larger). Equation (3) has similar characteristics in that survival decreases with age regardless of density but also decreases with increasing site index and density (which are factors that are expected to increase competition). Figure 6 is an example of survival curves for equation (3) without hurricane related mortality for breast height age 5 densities of 2000 and 600 trees per acre. Although thinning is not explicit in the model, the model will predict an increase in survival rate after thinning due to the decrease in trees per acre. Equation (3) is designed to provide logical long-term estimates of survival, and the residuals were centered near zero for both age and thinning classes (Figure 7). Although equation (3) captures the long-term trend in mortality and is biologically logical, mortality actually occurs in discrete steps and methods to improve survival equations based on stand structure should be pursued in the future.

This variable density model allows the incorporation of thinning and the estimation of quadratic mean dbh, which is a useful guide for evaluating dbh response to thinning. The ability to estimate trees per acre removed by thinning using ratio equation (7) or to estimate basal area removed by thinning using ratio equation (8) allows growth projections that incorporate the effect of thinning based on the thinning practices in

Figure 4. Boxplots of initial stand volume o.b. ($VOB_1$ top) and projected stand volume o.b. ($VOB_2$ bottom) residuals for equation system with $N_2$ known based on 1700 observations examined by age class (left) and target thinning basal area (right). Middle line is the median, dotted line is the mean, box edges are the 25th and 75th percentiles, whiskers are the 10th and 90th percentiles, and dots are the 5th and 95th percentiles.

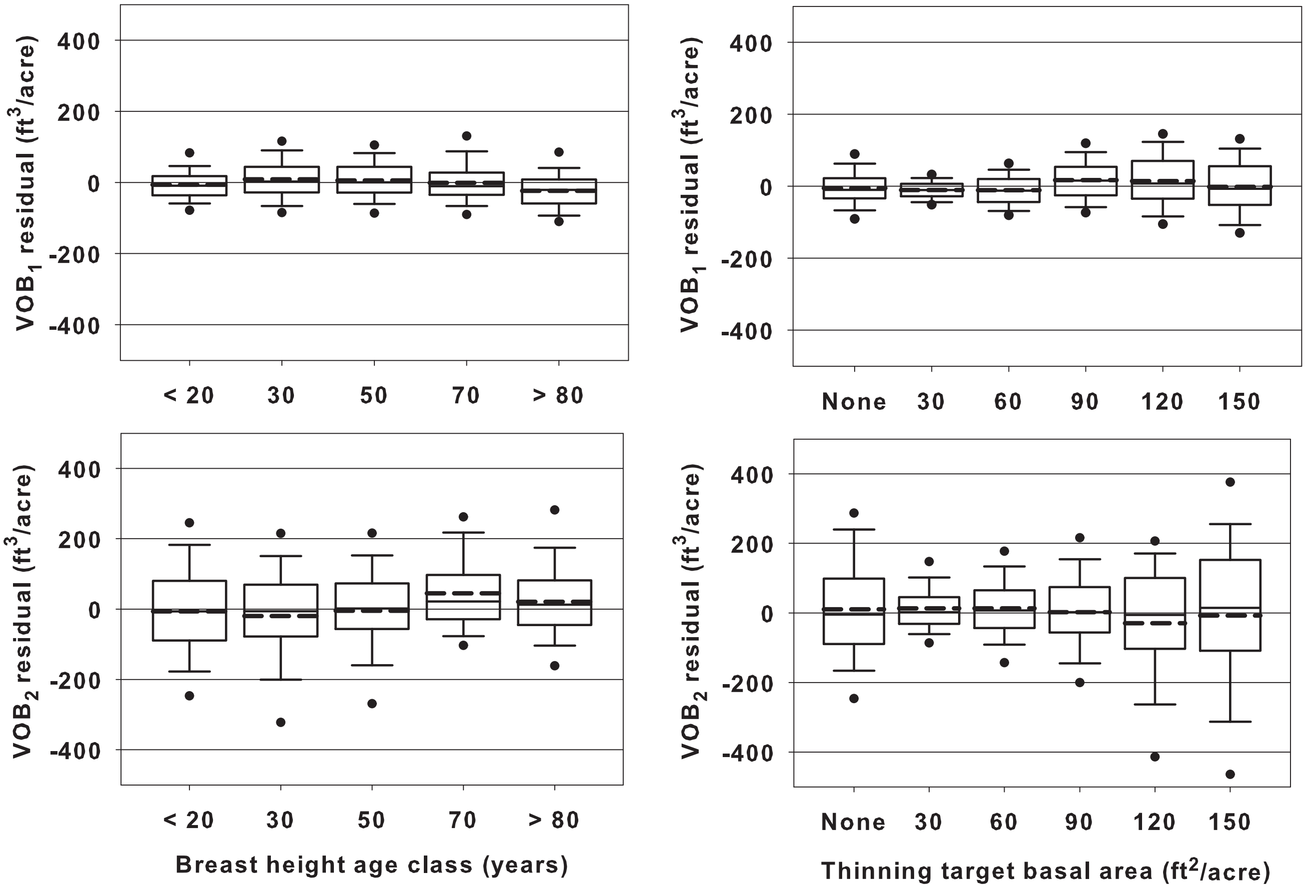


Figure 5. Boxplots for percent error of projected stand volume o.b. ($VOB_2$) residuals for equation system (4) with $N_2$ predicted using the survival model based on 1439 observations. Residuals are compared by age class (left) and target thinning basal area (right). Middle line is the median, dotted line is the mean, box edges are the 25th and 75th percentiles, whiskers are the 10th and 90th percentiles, and dots are the 5th and 95th percentiles.

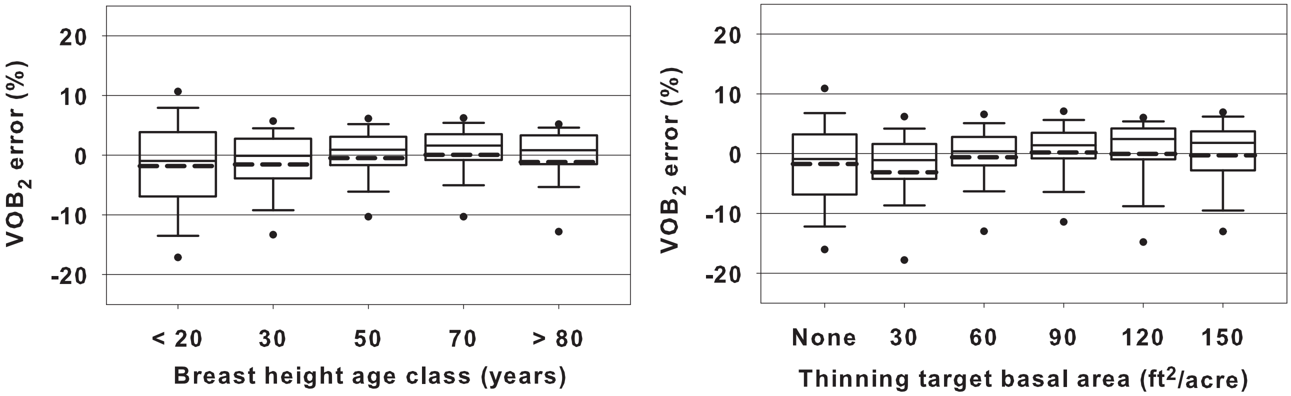
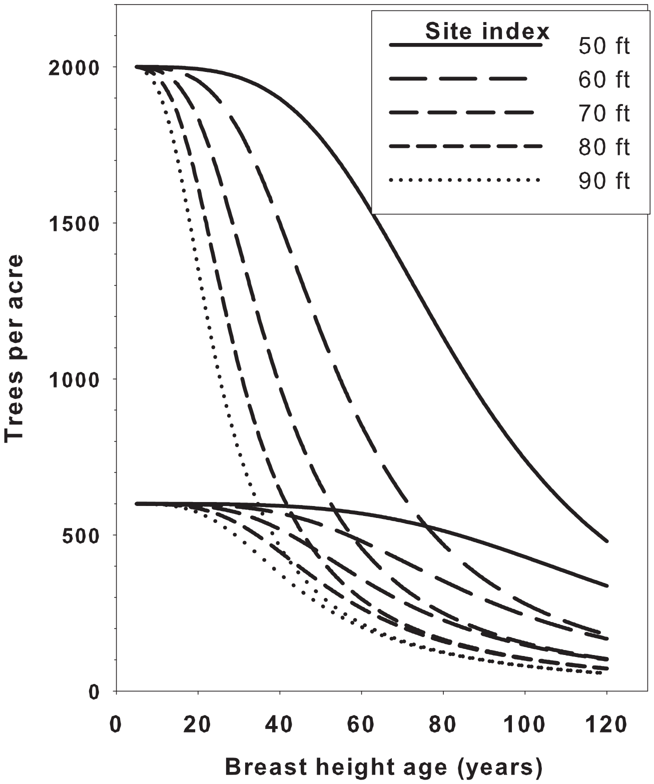


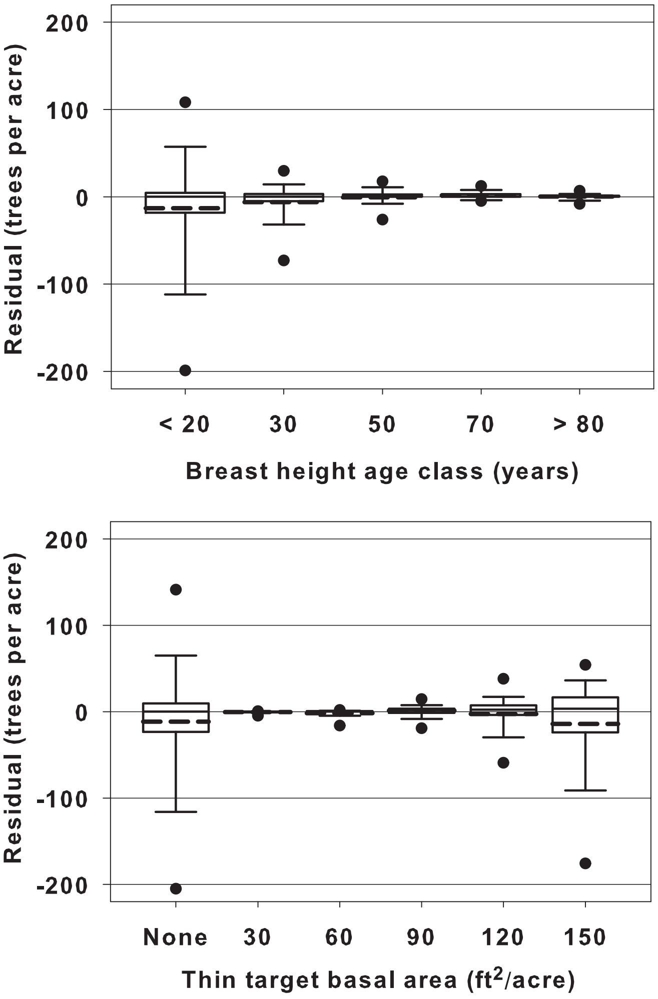
Figure 6. Survival equation (3) predictions for initial stand densities of 2000 and 600 trees per acre. Curves are shown for a range of site index represented in this dataset.



this study. This does not preclude different types of thinning, if basal area and trees per acre after thinning can be defined using an expected before- and after-thin stand table approach.

Prediction of merchantable volumes is noticeably absent from this system. A ratio approach similar to that of Amateis et al. (1986) was attempted but produced poor results because distribution of trees by dbh class is altered in so many unique ways by thinning. Farrar (1985) projected sawtimber basal area but this required an estimate of sawtimber basal area based on inventory data, which is essentially the same as requiring a stand table. The inability to estimate merchantable volumes demonstrates how important it is for this project to develop stand level models to predict stand growth with biological reasonable patterns as a precursor to tree level models that can distribute this growth at the tree level and improve mortality prediction. The integration of this stand level system with tree level models is required to produce an accurate modeling system that can predict the wide range of merchantable products that are produced over the long rotation length of longleaf pine.

Figure 7. Boxplots of residuals for survival equation (3) fit without hurricane mortality by breast height age class (top) and target thinning basal area (bottom). Middle line is the median, dotted line is the mean, box edges are the 25th and 75th percentiles, whiskers are the 10th and 90th percentiles, and dots are the 5th and 95th percentiles.



## Model Uses

The variable density component of this growth and yield model system provides the information required to manage natural stands under long rotations that often incorporate multiple thinnings. Comparison of future management options requires simulation and comparison of various scenarios dependent on management objectives. The use of the basal area projection equation in conjunction with survival curves provides improved information with regards to timing of thinning to anticipate and capitalize on potential mortality.

This dataset was impacted by hurricanes Frederick (1979), Opal (1995), Ivan (2004), and Katrina (2005). The survival curve was fit without hurricane damage and can be used for most forest management objectives if projection is made between hurricane salvages. The long-term impact of hurricanes on stands that are viable afterwards should be based on post-storm inventory. The inclusion of both stand basal area and trees per acre in projection models allows storm damage to be accounted for in projection models.

As with any statistical endeavor, this model has limitations. Data were limited for stands more than 90 years old. Site index ranged from 45 to 90 feet and estimates from the survival equation should be suspect at these limits and beyond. Although these stands received prescribed burning, more extreme burning regimes may decrease survival. Ideally, the ability to project a stand table, especially in repeatedly thinned stands, would be useful but that was beyond the scope of this project.

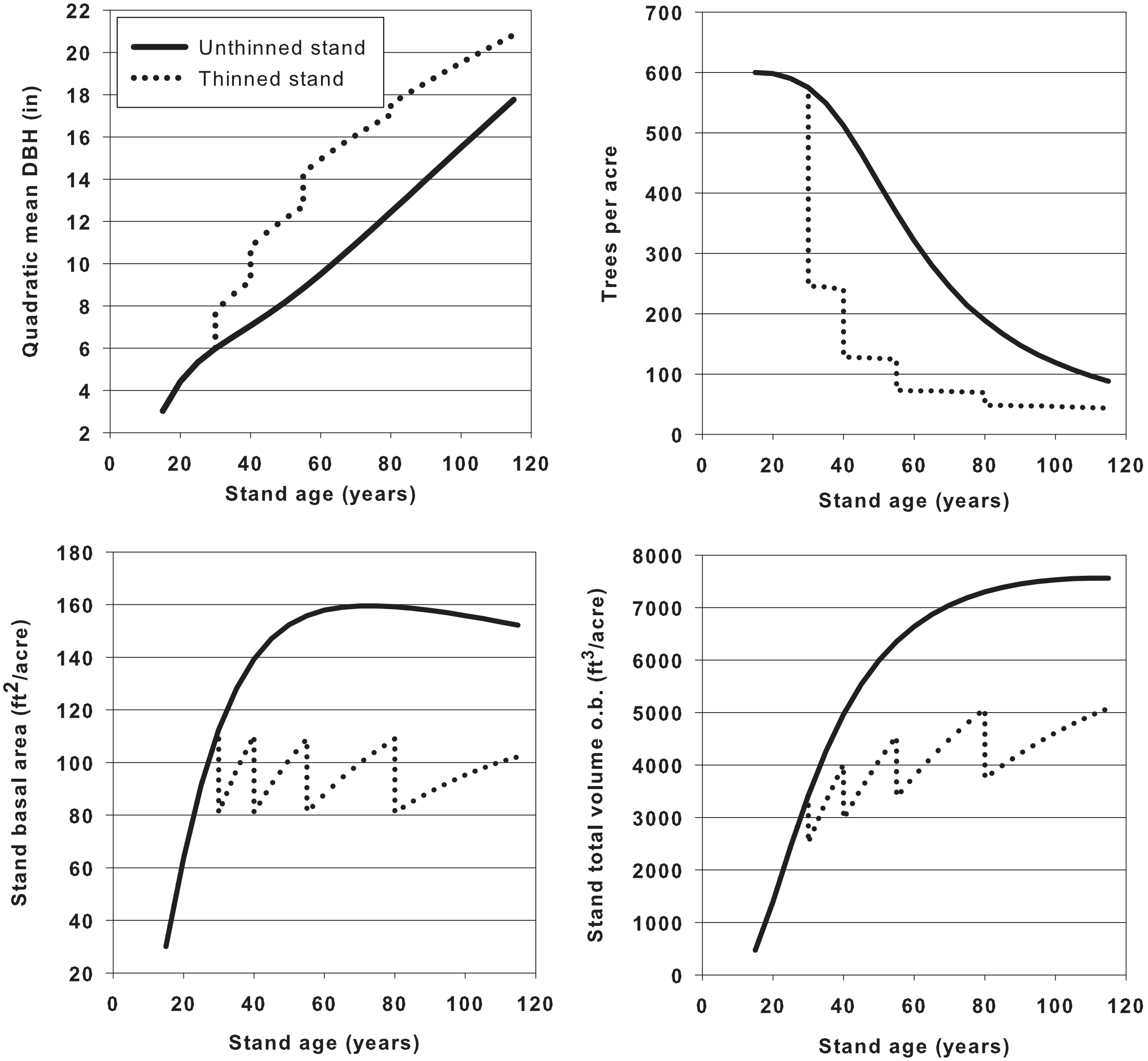
Growth and yield information for longleaf pine throughout its native range and for planted longleaf pine is limited. How well this model might be used outside the geographical range of the data is unclear. The application of this model requires testing in plantation stands more than 20 years old where site index, basal area, and tree density can be well characterized. There are structural differences between natural and planted stands. In this RLGS study of even-aged natural stands, only 25 percent of plots had a breast height age range of three years or less (Lauer and Kush 2010) as would be expected in plantation stands. The inclusion of the $\gamma_3$ parameter in equation (2), for instance, may be related to the breast height age range in these natural stands. Nonetheless, this model formulation uses breast height age for both site index and growth and yield equations and makes it possible to equate many of the stand parameters with those for plantation stands.

## GROWTH PREDICTION EXAMPLE

These models were used to predict growth to 115 years for a site index 80 stand that had 600 trees per acre and 30.1 square feet of basal area at stand age 15 (eight years breast height age). Predictions were compared with and without thinning. In this example, the thinning target was set at 90 square feet of basal area but thinning would not occur until this was exceeded by 20 percent and thinning was performed to 10 percent less than this target. In addition, thinning did not occur if quadratic mean dbh was less than 4.0 inches. Figure 8 shows the predictions in terms of quadratic mean dbh, trees per acre, stand basal area, and total volume o.b. Thinning had positive effects on quadratic mean dbh, which was 3.0 inches greater in the thinned stand at age 115. At stand age 60, average quadratic dbh was 14.9 versus 9.5 inches with and without thinning, respectively. Total volume o.b. for the thinned stand was 4,154 $ft^3$ $ac^{-1}$ with removals from previous thins of 3091 $ft^3$ $ac^{-1}$ for a total volume production of 7245 $ft^3$ $ac^{-1}$ compared to age 60 volume of 6641 $ft^3$ $ac^{-1}$ without thinning. This increase for the thinned stand is from captured mortality.

Long projection periods like this previous example can compare options but should be used with caution in terms of relating initial conditions at young ages to long-term outcomes of an individual stand. The survival equation in this system computes an average value and does not account for variation in actual conditions later in stand development. Predictions of individual stand growth beyond 15 years may be unreliable. This average system approach will drive long-term basal area predictions towards 150 square feet per acre. Actual inventories at older ages may find stands above this average that will project to higher basal areas. Long-term predictions will not accurately account for the distribution of individual stand outcomes. Nonetheless, patterns observed in Figure 8 follow those observed in the RLGS dataset. This model system summarizes forty years of invaluable experience with longleaf pine management in a way that is accessible to forest managers.

Figure 8. Comparison of quadratic mean dbh, trees per acre, stand basal area, and total volume o.b. predictions for a thinned and unthinned stand. Initial stand age was 15 years (8 breast height age) with 600 trees per acre, 30.1 square feet of basal area, and 80 site index. The basal area thinning target was 90 square feet. Stand was thinned when basal area thinning target was exceeded by 20 percent and thinned to 10 percent less than the target. Thinning did not occur if quadratic mean dbh was less than 4.0 inches.



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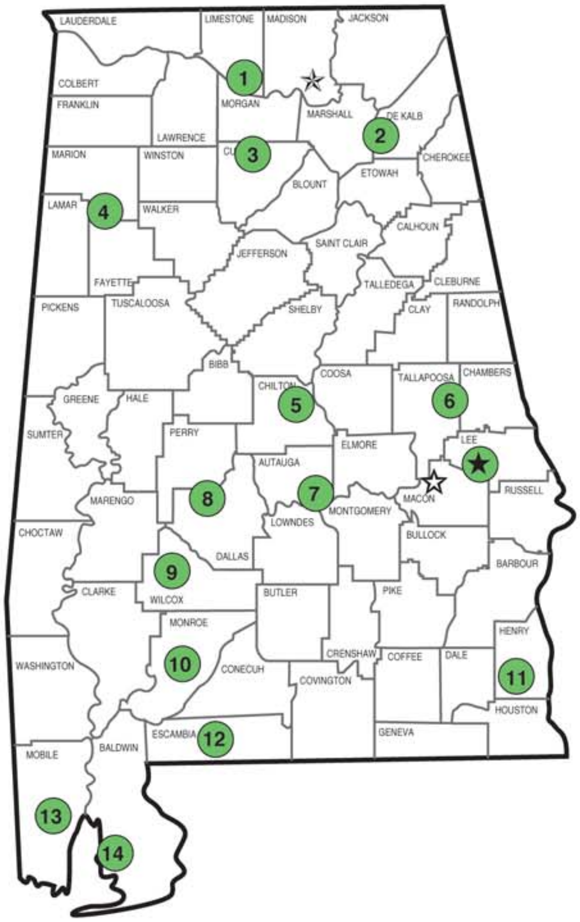

## Research Unit Identification

★ Main Agricultural Experiment Station, Auburn.
✯ Alabama A&M University.
☆ E. V. Smith Research Center, Shorter.

1. Tennessee Valley Research and Extension Center, Belle Mina.
2. Sand Mountain Research and Extension Center, Crossville.
3. North Alabama Horticulture Research Center, Cullman.
4. Upper Coastal Plain Agricultural Research Center, Winfield.
5. Chilton Research and Extension Center, Clanton.
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13. Ornamental Horticulture Research Center, Spring Hill.
14. Gulf Coast Research and Extension Center, Fairhope.